

Time evolution of the sQGP with hydrodynamic models

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Abstract The strongly interacting Quark-Gluon-Plasma (sQGP) created in relativistic nucleus-nucleus collisions, can be described by hydrodynamic models. Low energy hadrons are created after the so called freeze-out of this medium, thus their distributions reveal information about the final state of the sQGP. Photons are created throughout the evolution, so their distributions carry information on the whole expansion and cool down. We show results from an analytic, 1+3 dimensional perfect relativistic hydrodynamic solution, and compare hadron and photon observables to RHIC data. We extract an average equation of state of the expanding quark matter from this comparison. In the second part of this paper, we generalize the before mentioned analytic solution of relativistic perfect fluid hydrodynamics to arbitrary temperature-dependent Equation of State. We investigate special cases of this class of solutions, in particular, we present hydrodynamical solutions with an Equation of State determined from lattice QCD calculations.

1 Introduction

The almost perfect fluidity of the experimentally created strongly interacting Quark-Gluon-Plasma at the Relativistic Heavy Ion Collider (RHIC) [1] showed that relativistic hydrodynamic models can be applied in describing the space-time picture of heavy-ion collisions and infer the relation between experimental observables and the initial conditions.

In this paper we investigate the relativistic, ellipsoidally symmetric model of Ref. [2]. Hadronic observables were calculated in Ref. [3], while photonic observables in Ref. [4]. We also show new solutions, which can be regarded as generalizations of the model of Ref. [2] to arbitrary, temperature dependent speed of sound, originally published in Ref. [5].

2 Equations of hydrodynamics

We denote space-time coordinates by $x^\mu = (t, \mathbf{r})$, with $\mathbf{r} = (r_x, r_y, r_z)$ being the spatial three-vector and t the time in lab-frame. The metric tensor is $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Coordinate proper-time is defined as $\tau = \sqrt{t^2 - |\mathbf{r}|^2}$. The fluid four-velocity is $u^\mu = \gamma(1, \mathbf{v})$, with \mathbf{v} being the three-velocity, and $\gamma = 1/\sqrt{1 - |\mathbf{v}|^2}$. An analytic hydrodynamical solution is a functional form for pressure p , energy density ε , entropy density σ , temperature T , and (if the fluid consists of individual conserved particles, or if there is some conserved charge or number) the conserved number density is n . Then basic hydrodynamical equations are the continuity and energy-momentum-conservation equations:

$$\partial_\mu (nu^\mu) = 0 \quad \text{and} \quad \partial_\nu T^{\mu\nu} = 0. \quad (1)$$

The energy-momentum tensor of a perfect fluid is

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - pg^{\mu\nu}. \quad (2)$$

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The energy-momentum conservation equation can be then transformed to (by projecting it orthogonal and parallel to u^μ , respectively):

$$(\varepsilon + p) u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu) \partial_\nu p, \quad (3)$$

$$(\varepsilon + p) \partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon = 0. \quad (4)$$

Eq. (3) is the relativistic Euler equation, while Eq. (4) is the relativistic form of the energy conservation equation. Note also that Eq. (4) is equivalent to the entropy conservation equation:

$$\partial_\mu (\sigma u^\mu) = 0. \quad (5)$$

The Equation of State (EoS) closes the set of equations. We investigate the following EoS:

$$\varepsilon = \kappa(T) p, \quad (6)$$

while the speed of sound c_s is calculated as $c_s = \sqrt{\partial p / \partial \varepsilon}$, i.e. for constant κ , the relation $c_s = 1/\sqrt{\kappa}$ holds. For the case when there is a conserved n number density, we also use the well-known relation for ideal gases:

$$p = nT. \quad (7)$$

For $\kappa(T) = \text{constant}$, an ellipsoidally symmetric solution of the hydrodynamical equations is presented in Ref. [2]:

$$u^\mu = \frac{x^\mu}{\tau}, \quad n = n_0 \frac{V_0}{V} \nu(s), \quad T = T_0 \left(\frac{V_0}{V} \right)^{\frac{1}{\kappa}} \frac{1}{\nu(s)}, \quad V = \tau^3, \quad s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}, \quad (8)$$

where n_0 and T_0 correspond to the proper time when the arbitrarily chosen volume V_0 was reached (i.e. $\tau_0 = V_0^{1/3}$), and $\nu(s)$ is an arbitrary function of s . The quantity s has ellipsoidal level surfaces, and obeys $u^\nu \partial_\nu s = 0$. We call s a *scaling variable*, and V the effective volume of a characteristic ellipsoid. Furthermore, X , Y , and Z are the time (lab-frame time t) dependent principal axes of an expanding ellipsoid. They have the explicit time dependence as $X = \dot{X}_0 t$, $Y = \dot{Y}_0 t$, and $Z = \dot{Z}_0 t$, with \dot{X}_0 , \dot{Y}_0 , \dot{Z}_0 constants.

3 Photon and hadron observables for constant EoS

From the above hydrodynamic solution with a constant EoS, source functions can be written up. For bosonic hadrons, it takes the following form [3]:

$$S(x, p) d^4x = \mathcal{N} \frac{p_\mu d^3 \Sigma^\mu(x) H(\tau) d\tau}{n(x) \exp(p_\mu u^\mu(x)/T(x)) - 1}, \quad (9)$$

where $\mathcal{N} = g/(2\pi)^3$ (with g being the degeneracy factor), $H(\tau)$ is the proper-time probability distribution of the freeze-out. It is assumed to be a δ function or a narrow Gaussian centered at the freeze-out proper-time τ_0 . Furthermore, $\mu(x)/T(x) = \ln n(x)$ is the fugacity factor and $d^3 \Sigma_\mu(x) p^\mu$ is the Cooper-Frye factor (describing the flux of the particles), and $d^3 \Sigma_\mu(x)$ is the vector-measure of the freeze-out hyper-surface, pseudo-orthogonal to u^μ . Here the source distribution is normalized such as $\int S(x, p) d^4x d^3 \mathbf{p} / E = N$, i.e. one gets the total number of particles N (using $c=1$, $\hbar=1$ units). Note that one has to change variables from τ to t , and so a Jacobian of $d\tau/dt = t/\tau$ has to be taken into account.

For the source function of photon creation we have [4]:

$$S(x, p) d^4x = \mathcal{N}' \frac{p_\mu d^3 \Sigma^\mu(x) dt}{\exp(p_\mu u^\mu(x)/T(x)) - 1} = \mathcal{N}' \frac{p_\mu u^\mu}{\exp(p_\mu u^\mu(x)/T(x)) - 1} d^4x \quad (10)$$

where $p_\mu d^3 \Sigma^\mu$ is again the Cooper-Frye factor of the emission hyper-surfaces. Similarly to the previous case, we assume that the hyper-surfaces are pseudo-orthogonal to u^μ , thus $d^3 \Sigma^\mu(x) = u^\mu d^3x$. This yields then $p_\mu u^\mu$ which is the energy of the photon in the co-moving system. The photon creation is assumed to happen from an initial time t_i until a point sufficiently near the freeze-out. From these source functions, observables can be calculated, as detailed in Refs. [3, 4]

Dataset		N_1 and HBT hadrons	elliptic flow hadrons	N_1 photons
Central FO temperature	T_0 [MeV]	199 ± 3	204 ± 7	204 MeV (fixed)
Eccentricity	ϵ	0.80 ± 0.02	0.34 ± 0.03	0.34 (fixed)
Transverse expansion	u_t^2/b	-0.84 ± 0.08	-0.34 ± 0.01	-0.34 (fixed)
FO proper-time	τ_0 [fm/c]	7.7 ± 0.1	-	7.7 (fixed)
Longitudinal expansion	\hat{Z}_0^2/b	-1.6 ± 0.3	-	-1.6 (fixed)
Equation of State	κ	-	-	7.9 ± 0.7

Table 1. Parameters of the model determined by hadron and photon observables data. See details in Refs. [3, 4].

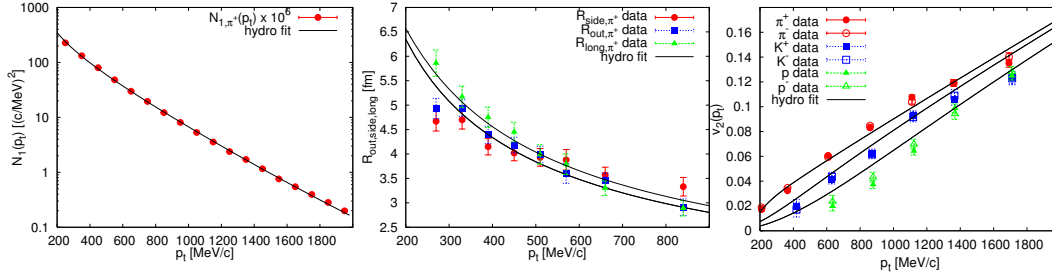


Figure 1. Fits to invariant momentum distribution of pions [6] (left), HBT radii [7] (middle) and elliptic flow [8] (right). See the obtained parameters in Table 1.

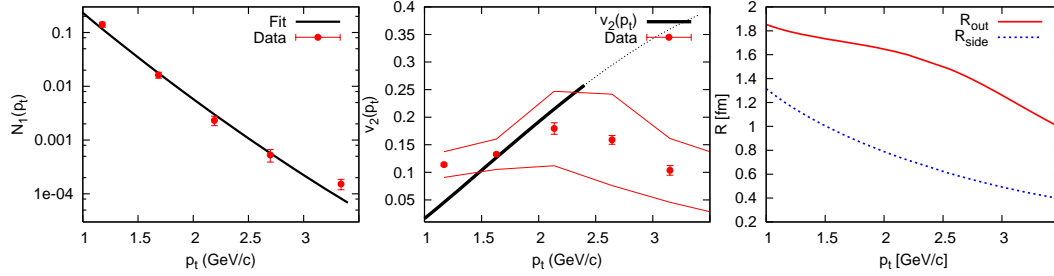


Figure 2. Fit to direct photon invariant transverse momentum data [9] (left), comparison to elliptic flow data [10] (middle) and direct photon HBT predictions (right). See the model parameters in Table 1.

4 Comparison to measured hadron and photon distributions

Observables calculated from the above source functions were compared to data in Refs. [3, 4]. Hadron fits determined the freeze-out parameters of the model [3]: expansion rates, freeze-out proper-time and freeze-out temperature (in the center of the fireball). When describing direct photon data [4], the free parameters (besides the ones fixed from hadronic fits) were κ (the equation of state parameter) and t_i , the initial time of the evolution.

We compared our model to PHENIX 200 GeV Au+Au hadron and photon data from Refs. [6–9]. Results are shown in Figs. 1 and 2, while the model parameters are detailed in Table. 1. The EoS result from the photon fit is $\kappa = 7.9 \pm 0.7_{stat} \pm 1.5_{syst}$, or alternatively, using $\kappa = 1/c_s^2$

$$c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst} \quad (11)$$

which is in agreement with lattice QCD calculations [11] and measured hadronic data [12, 13]. This represents an average EoS as it may vary with temperature. The maximum value for t_i within 95% probability is 0.7 fm/c. The initial temperature of the fireball (in its center) is then:

$$T_i = 507 \pm 12_{stat} \pm 90_{syst} \text{ MeV} \quad (12)$$

at 0.7 fm/c. This is in accordance with other hydro models as those values are in the 300 – 600 MeV interval [9]. Note that the systematic uncertainty comes from the analysis of a possible prefactor, as detailed in Ref. [4].

Using the previously determined fit parameters, we can calculate the elliptic flow of direct photons in Au+Au collisions at RHIC. This was compared to PHENIX data [10], as shown in Fig. 2, and they were found not to be incompatible. We also calculated direct photon HBT radii as a prediction, and found R_{out} to be significantly larger than R_{side} .

5 New solutions for general Equation of State

We found new solutions to the relativistic hydrodynamical equations for arbitrary $\varepsilon = \kappa(T) p$ Equation of State, as detailed in Ref. [5]. These are the first solutions of their kind (i.e. with a non-constant EoS). In the case where we do not consider any conserved n density, the solution is given as:

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \quad (13)$$

$$u^\mu = \frac{x^\mu}{\tau}, \quad (14)$$

$$\frac{\tau_0^3}{\tau^3} = \exp \left\{ \int_{T_0}^T \left(\frac{\kappa(\beta)}{\beta} + \frac{1}{\kappa(\beta) + 1} \frac{d\kappa(\beta)}{d\beta} \right) d\beta \right\}. \quad (15)$$

For the case when the pressure is expressed as $p = nT$ with a conserved density n , another new solution can be written up as:

$$n = n_0 \frac{\tau_0^3}{\tau^3}, \quad (16)$$

$$u^\mu = \frac{x^\mu}{\tau}, \quad (17)$$

$$\frac{\tau_0^3}{\tau^3} = \exp \left\{ \int_{T_0}^T \left(\frac{1}{\beta} \frac{d}{d\beta} [\kappa(\beta)\beta] \right) d\beta \right\}. \quad (18)$$

Quantities denoted by the subscript 0 (n_0 , T_0 , σ_0) correspond to the proper-time τ_0 , which can be chosen arbitrarily. If for example τ_0 is taken to be the freeze-out proper-time, then T_0 is the freeze-out temperature. These solutions are simple generalizations of the $v(s) = 1$ case of the solutions of Ref. [2], and the latter also represents a relativistic generalization of the solution presented in Ref. [14].

It is important to note that the conserved n solution becomes ill-defined, if $\frac{d}{dT} (\kappa(T) T) > 0$ is not true. In such a case, one can use the solution without conserved n (Eqs. (13)–(15)).

If κ is given as a function of the pressure p and not that of the temperature T , a third new solution can be given as:

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \quad (19)$$

$$u^\mu = \frac{x^\mu}{\tau}, \quad (20)$$

$$\frac{\tau_0^3}{\tau^3} = \exp \left\{ \int_{p_0}^p \left(\frac{\kappa(\beta)}{\beta} + \frac{d\kappa(\beta)}{d\beta} \right) \frac{d\beta}{\kappa(\beta) + 1} \right\}, \quad (21)$$

i.e. almost the same as in Eq. (15), except that here the integration variable is the pressure p .

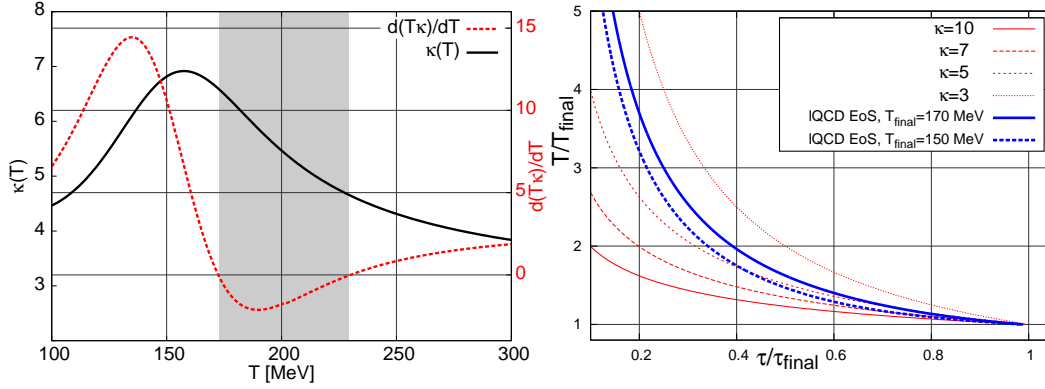


Figure 3. Left: The temperature dependence of the EoS parameter κ from Ref. [11] is shown with the solid black curve. In the shaded T range (173 MeV - 230 MeV) $\frac{d}{dT}(\kappa(T)T)$ (red dashed line) becomes negative, thus the solution shown in Eqs. (13)–(15) shall be used with this EoS. Right: Time dependence of the temperature $T(\tau)$ (normalized with the freeze-out time τ_f and the freeze-out temperature T_f) is shown. The four thin red lines show this dependence in case of constant κ values, while the thicker blue lines show results based on the EoS of Ref. [11].

6 Utilizing a lattice QCD EoS

A QCD equation of state has been calculated by the Budapest-Wuppertal group in Ref. [11], with dynamical quarks, in the continuum limit. In their Eq. (3.1) and Table 2, they give an analytic parametrization of the trace anomaly $I = \epsilon - 3p$ as a function of temperature. The pressure can also be calculated from it, as (if using the normalized values and $\hbar = c = 1$ units) $\frac{I}{T^4} = \frac{1}{T} \frac{\partial}{\partial T} \frac{p}{T^3}$. From this, we calculated the EoS parameter $\kappa = I/p + 3$ as a function of the temperature, as shown in Fig. 3 (left plot). Since in a T range $\frac{d}{dT}(\kappa(T)T)$ becomes negative, the solution without conserved number density n (presented in Eqs. (13)–(15)) was used. [5]

We utilized the obtained $\kappa(T)$ and calculated the time evolution of the temperature of the fireball from this solution of relativistic hydrodynamics. The result is shown in Fig. 3 (right plot). Clearly, temperature falls off almost as fast as in case of a constant $\kappa = 3$, an ideal relativistic gas. Hence a given freeze-out temperature yields a significantly higher initial temperature than a higher κ (i.e. a lower speed of sound c_s) would.

Let us give an example! We shall fix the freeze-out temperature, based on lattice QCD results, to a reasonable value of $T_f = 170$ MeV. Let all the quantities with subscript 0 correspond to the freeze-out, we shall thus index them with f . In this case, already at $0.3 \times \tau_f$ (30% of the freeze-out time), temperatures 2.5-3 \times higher than at the freeze-out can be reached. To give a full quantitative example, let us assume the following values:

$$\tau_f = 8 \text{ fm}/c \text{ and } \tau_{\text{init}} = 1.5 \text{ fm}/c, \text{ then} \quad (22)$$

$$T_f = 170 \text{ MeV} \Rightarrow T_{\text{init}} \approx 550 \text{ MeV} \quad (23)$$

(and even higher if τ_{init} is smaller). This value would have been reached with a constant EoS of $\kappa \approx 4$, even though the extracted average EoS values are usually above this value. The reason for it may be, that for the largest temperature range, κ obtains values close to 4, as shown in the left plot of Fig. 3.

In general, the mentioned lattice QCD equation of state of Ref. [11] and our hydro solution yields a $T(\tau)$ dependence. Then, if the freeze-out temperature T_f and the time evolution duration $\tau_f/\tau_{\text{init}}$ are known, the initial temperature of the fireball can be easily calculated, or even read off the right plot of Fig. 3, as it was drawn with units normalized by the freeze-out temperature and proper-time.

7 Conclusion

Exact parametric solutions of perfect hydrodynamics can be utilized in order to describe the matter produced in heavy ion collisions at RHIC. We calculated observables from a relativistic, 1+3 dimensional, ellipsoidally symmetric, exact solution, and compared these to 200 GeV Au+Au PHENIX data. Hadronic data are compatible with our model, and freeze-out parameters were extracted from fits to these data. [3]

From fits to direct photon data, we find that thermal radiation is consistent these measurements, with an average speed of sound of $c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst}$. We can also set a lower bound on the initial temperature of the sQGP to $507 \pm 12_{stat} \pm 90_{syst}$ MeV at 0.7 fm/c. We also find that the thermal photon elliptic flow from this mode is not incompatible with measurements. We also predicted photon HBT radii from this model. [4]

In the second part of this paper, we have presented the first analytic solutions of the equations of relativistic perfect fluid hydrodynamics for general temperature dependent speed of sound (ie. general Equation of State). Using our solutions and utilizing a lattice QCD Equation of State, we explored the initial state of heavy-ion reactions based on the reconstructed final state. In $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions, our investigations reveal a very high initial temperature consistent with calculations based on the measured spectrum of low momentum direct photons. [5]

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